Trade-off Analysis in Multi-objective Optimization Using Chebyshev Orthogonal Polynomials

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In this paper, it is intended to introduce a method to solve multi-objective optimization problems and to evaluate its performance. In order to verify the performance of this method it is applied for a vertical roller mill for Portland cement. A design process is defined with the compromise decision support problem concept and a design process consists of two steps : the design of experiments and mathematical programming. In this process, a designer decides an object that the objective function is going to pursuit and a non-linear optimization is performed composing objective constraints with practical constraints. In this method, response surfaces are used to model objectives (stress, deflection and weight) and the optimization is performed for each of the objectives while handling the remaining ones as constraints. The response surfaces are constructed using orthogonal polynomials, and orthogonal array as design of experiment, with analysis of variance for variable selection. In addition, it establishes the relative influence of the design variables in the objectives variability. The constrained optimization problems are solved using sequential quadratic programming. From the results, it is found that the method in this paper is a very effective and powerful for the multi-objective optimization of various practical design problems. It provides, moreover, a reference of design to judge the amount of excess or shortage from the final object.

Key Words : Trade-off Analysis, Multi-objective Optimization, Response Surface Method (RSM), Design of Experiments (DOE), Chebyshev Orthogonal Polynomial

1. Introduction

In these days, reduction of design processes, design time and higher design quality have been typically required for the engineering design optimization problems. They have been solved rapidly during recent years, mainly due to faster computation capability, better algorithms, more fre-

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quent use of finite element analysis (FEA) and mathematical programming (Zhang et al., 2003). Most methods in the literatures, however, are applicable to only single-objective optimization problems. In addition, in these single-objective methods, differentiability is required basically and this limits their applicability to more general problems. Gunawan and Azarm (2005) developed a robust optimization method that addresses some of the shortcomings of the previous methods. Despite its advantages, however, Gunawan and Azarm's method is applicable only to single-objective optimization problems.

For complex problems, such as multi-objective problems, the objective functions are often noisy and it is hard to find their gradients. There are few attempts in the literature for robust optimization problems that have multiple criteria. The attempts followed the Taguchi's design of experiment (DOE) approach: Pignatiello (1993), Elsayed and Chen (1993), Tsui (1999). Those method are hard to obtain better solutions because of limited number of candidate design points. To overcome this defect, it is necessary to fit objective function to design variables by using response surface methodology (RSM) (Roux et al., 1998; Timothy et al., 2001; Youn et al., 2004; Redhe et al., 2004). By the introduction of these response surfaces, noisy or unphysical components of the response would be smoothed out. The optimal solution is then searched on these fitted response surfaces rather than on the real ones. This optimization depends on the proper level and arrangement of design variables because the orthogonal arrays are repeatedly used during design processes. In addition, the design space between the level of design variables can not be accurately represented by the fitted response surface. Thus mathematical programming is required for better optimal solutions after large reduction of an objective function by DOE approach (Kurtaran et al., 2002).

Sequential Quadratic Programming (SQP) is suitable for continuous nonlinear objective functions such as stress, deflection and natural frequency and so on, with both equality and inequality constraint functions. SQP is much more practical than any other similar algorithms (Redhe et al., 2004) since the convergence is typically achieved in a few iterations.

It is attempted, in this study, to express the objective functions with design variables after performing structural analysis following the design of experiment. In addition, it is performed to obtain the relationship between the objective functions and the design variables by the use of F-test. And then, the SQP is applied for structural multi-objective optimization problem of portland cement vertical roller mill and the best optimal solution is obtained from the sets of Pareto solutions.

2. Background and Proposed Methodology

In this study, a combination of the RSM and SQP has been used. Detailed optimization process will be presented in the following chapter.

Figure 1 shows the procedure of multi-objective optimization using DOE, RSM and SQP. As seen in the figure, the first step is to identify the design space. This is typically a multi-dimensional problem defined by the upper and lower bounds of each design variable over some region of interest. Once the design space has been identified, an experimental design is selected to sam-

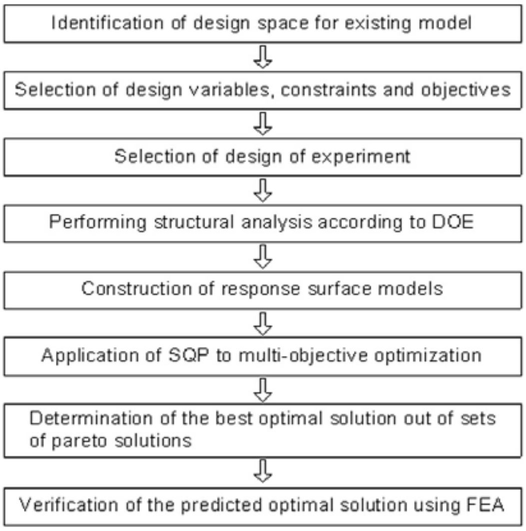

Fig. 1 Procedure of multi-objective optimization

ple the design space. After an experimental design is chosen, the design space is sampled to construct approximations of each objective and constraint function. In this study, two approximations (Chen et al., 2002) are suggested.

(1) First-order multiple regression polynomial equation

(2) Second-order Chebyshev orthogonal polynomial equation

After the approximations have been constructed, they must be verified to ensure sufficient accuracy. Verification can be achieved through mean absolute percentage error.

$$\text{MAPE}=\frac{1}{n}\sum_{i=1}^{n}|y_i-\bar{y}_i|\times 100 \tag{1}$$

where y_i is the actual value of the objective function, $\bar{y}_i$ is the predicted value of the objective function, and n is the number of data. Once the response surface model has been verified, SQP can be applied to design points. And then the best optimal solution is selected and this predicted optimal solution is verified by FEA.

2.1 Response surface method

Design of experiments is performed prior to any FEA. The difficulty lies in minimizing the number of computer simulations, at the same time, in obtaining a good response surface approximation. To carry out response surface method, the regression method is applied to FEA results to build mathematical models. The models are then formulated as an objective function in an optimization problem that is consequently optimized using SQP to obtain the minimum mass, stress and deflection of vertical roller mill. The RSM is used to fit the FEA data to the multiple regression polynomial equation of degree one and Chebyshev orthogonal polynomial equation of degree two to obtain regression coefficients. The former is obtained by MINITAB R.13, while the latter is obtained by Excel. The following Eqs. (2) and (3) denote the multiple regression polynomial equation and Chebyshev orthogonal polynomial equation, respectively (Park, 1996).

$$y=\beta_0+\beta_1 x_1+\beta_1 x_1\cdots+\beta_i x_i+e \tag{2}$$

where y represents mass, stress and deflection, x_i : the i-th design variable, β_i : the i-th estimated regression coefficient, e : experimental error in each model.

Using Chebyshev orthogonal polynomial, the levels of each design variable are assumed to be equally spaced in Eq. (3)

$$\begin{aligned} y=&\beta_0+\beta_1(x-\bar{x})+\beta_2\{(x-\bar{x})-c^2(l^2-1)/12\}\\ &+\beta_3\{(x-\bar{x})^3-c^2(3l^2-7)(x-\bar{x})/20\}\\ &+\cdots+\beta_k P_k(x)\cdots \end{aligned} \tag{3}$$

A degree k should be less than a level number l and maximum degree of design variable is l-1. $\bar{x}$ is the average of the design variable values and c is the level interval coefficient. β_k are given as an orthogonality coefficient in Eq. (4) (Gautschi, 1996 ; Baek et al., 2004).

β_0=The average of all the FEA values

$$\beta_i=b_i\sum_{v=1}^{a}\xi_i(A_v)\,y_v/\sum_{v=1}^{a}\xi_i^2(A_v)\,y_v \tag{4}$$

where A_v means each level of A and y_v means the average of the FEA values at each level. Taylor series approximation is not considered as an objective function because it is suitable only for narrow level interval of design variable. An approximate equation in this study needs wider level interval.

Chebyshev orthogonal polynomial, compared with Taylor series is available for sufficiently wide level interval of design variables. It is thought, therefore, that Chebyshev orthogonal polynomial is very effective for the case that the level interval of design variables is very wide like the practical structure design. The accuracy of predicted optimal solution is verified by FEA.

2.2 Design of experiment

In order to minimize the effect of the noise on the fitted polynomial and to improve the presentation of the design space, the design of experiment can be used to select the data used in the construction of response surfaces. There are a number of different design of experiment techniques. In this study, orthogonal array (OA) is used to select an effective design space. OA is a fractional factorial matrix that assures a balanced comparison of levels of any factors or interaction of factors. Because the points are not necessarily at vertices, orthogonal array tools can be more robust than any other design of experiments. Based on the theory of design of experiments, OA can significantly reduces the number of experimental configurations.

2.3 Optimization algorithms using SQP

Engineering design by its very nature is multi-objective, often requiring trade-offs between disparate and conflicting objectives. Designing the cross-section of a cantilever beam is a classic example of the trade-offs embodied in design : minimizing the weight and deflection of the beam requires a trade-off between both objectives since improving one worsens the other. Thus multi-objective design problems are dependent on how a designer treats the priority of objective functions (Bramanti et al., 2001 ; Wilson et al., 2001 ; Tappeta et al., 2001).

Generally, objective functions are divided into two kinds : one objective function which must be satisfied and the other objective functions which is treated as behavior constraints. For a single objective function an optimization is performed repeatedly from high to low priority. The basic principle of SQP is to replace the given nonlinear problem by a sequence of quadratic subproblems that are easier to solve. Proper convergence properties of constraints are achieved with some modifications on the basis of SQP algorithms. An SQP procedure implemented in the this study (MATLAB software) is employed to minimize mass, stress and deflection that are formulated by RSM, a polynomial functions of some design variables bounded by upper and lower limits. The basic process of an SQP can be expressed in the following steps.

Step 1 : Set up and solve a quadratic programming (QP) subproblem, giving a search direction

Step 2 : Test for convergence, stop if it is satisfied.

Step 3 : Step forward to a new point along the search direction.

Step 4 : Update the hessian matrix in QP and go to step 1

2.4 Model verification

After optimal conditions are obtained by the SQP, experiments based on the conditions are performed by FEA. The results are then analyzed by analysis of variance (ANOVA) from MINITAB R.13, with F-test to detect the difference between the predicted values and the observed ones.

2. Industrial Application

3.1 Vertical roller mill

A vertical roller mill consists of 8 table liners with radius 2.39 m. Its shape is symmetrical about the center axis and its volume is $16{,}059 \times 10^6$ m^3. It revolves stably at 2.34 rad/s. A general finite element analysis program used in this study is ANSYS. The 3-D structural model is composed of 100,638 nodes and 88,149 finite elements. The results of structural analysis of Baek et al.(2004) is referred in this analysis.

Figure 2 shows the operation state of vertical roller mill and the data are gathered in every 4 seconds and printed in every 10 minutes. It seems that the vertical roller mill is operated stably for

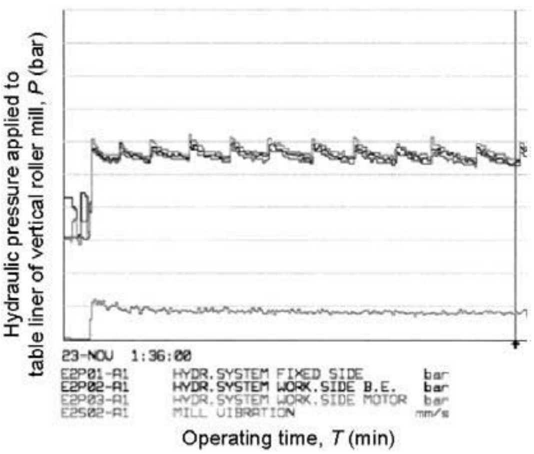

Fig. 2 Normal operating conditions for vertical roller mill

repeated load conditions. These results provide quantitative information to decide the objective and constraint functions of structural behavior of the mill.

Based on the above information, 7 design variables are chosen as shown in Fig. 3 to reinforce the area of the mill where the stress is concentrated.

Figure 4 shows the stress and deflection distribution of the primitive design model. It is important to control the stress, deflection and mass of the mill from the viewpoint of the safety and productivity. It is attempted to establish objective functions of the mill with stress, deflection and weight of the table liner.

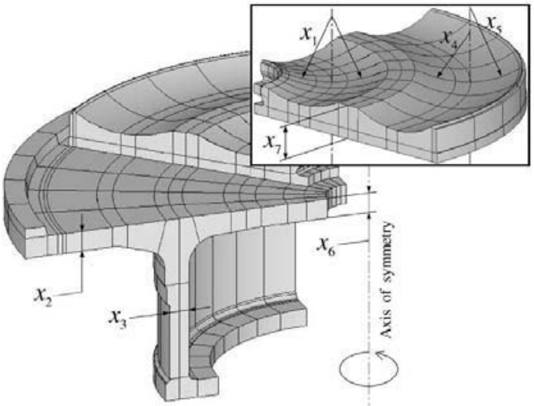

Fig. 3 FE model and design variables of vertical roller mill

3.2 Response surface modeling

It is intended in this study to develope models representing the mass, stress, deflection of a vertical roller mill using RSM. In Table 1 levels of design variables are shown. Mixed orthogonal array of $L_{18}(2^1 \times 3^7)$ is used to perform DOE because there are design variables with two different levels, and interactions can be ignored. Using this experimental design, the levels of each design variable are assumed to be equally spaced. The FEA results of their means based on OA and ANOVA are presented in Tables 2, 3 and 4.

Significance probability P is a very useful parameter which determines the significance of design variable. If the value of P is less than 0.01, it is said that a design variable is significant at the 0.01 or 1% level. According to the results, approximate equations are constructed by a curve

Table 1 The uncoded design variables and their levels

Design variable	Level 1	Level 2	Level 3
x_7	initial	baseline	—
x_1	baseline	15 mm	25 mm
x_2	−10 mm	−20 mm	baseline
x_3	−40 mm	−20 mm	baseline
x_4	baseline	20 mm	40 mm
x_5	−40 mm	−20 mm	baseline
x_6	baseline	20 mm	40 mm

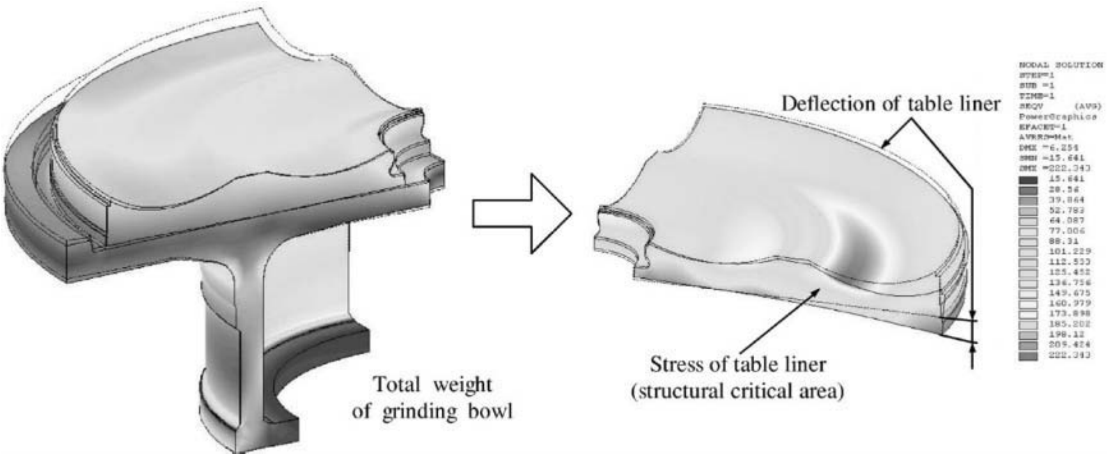

Fig. 4 Stress distribution of the primitive model

Table 2 Result of structural analysis

Analysis number	Mass (kg/mm^3)	Table liner stress (MPa)	Deflection (mm)
1	1.1463	214.083	5.817
2	1.1961	193.521	5.211
3	1.2771	180.240	4.684
4	1.2160	191.223	5.330
5	1.2116	187.398	5.069
6	1.2363	205.432	4.928
7	1.2122	207.640	5.165
8	1.2621	189.614	4.653
9	1.2191	188.833	5.295
10	1.2325	180.974	4.756
11	1.1579	202.583	5.588
12	1.1832	197.703	5.267
13	1.2419	179.587	4.867
14	1.1835	209.993	5.050
15	1.1926	195.624	5.395
16	1.2207	198.468	4.821
17	1.1997	183.535	5.219
18	1.2268	201.105	5.031

Table 3 Variance analysis of multiple regression polynomial for deflection

Design variable	Unstandardized		Std. coeff.	T-ratio	P-value
	B	Std. error	Beta		
Const.	6.467	0.044	—	147.513	0
x_7	−1.8E-2	0.013	−0.029	−1.358	0.204
x_1	−9.5E-2	0.008	−0.526	−11.989	0
x_2	−1.3E-2	0.008	−0.035	−1.642	0.132
x_3	−0.313	0.008	−0.845	−39.493	0
x_4	−0.141	0.008	−0.380	−17.778	0
x_5	−2.4E-3	0.008	−0.007	−0.305	0.766
x_6	−9.7E-2	0.008	−0.263	−12.273	0
Model	Sum of squares	DOF	Mean square	F-ratio	P-value
Regression	1.636	7	0.234	310.674	0
Residual	7.552E-2	10	7.522E-4		
Total	1.643	17			

fitting procedure. In case of multiple regression polynomial adequate models are selected from all the design variables while in case of Chebyshev orthogonal polynomial is selected only from statistically significant design variables. In this study the regression coefficients of statistically significant models are given in Eqs. (5) and (6)

$$y^{1st}_{deflection}=6.467-0.018x_1-0.095x_2-0.013x_3 -0.313x_4-0.141x_5-0.0024x_6-0.097x_7 \quad (5)$$

$$y^{2nd}_{deflection}=6.498-0.0949x_2-0.418x_4 +0.02633x_4^2-0.1406x_5^2-0.09717x_7 \quad (6)$$

The adequacy and the fitness of the functions of design variables of response surfaces modeled are examined using ANOVA.

In Table 5 the mean absolute percentage error for each model (polynomial) is shown to compare FEA values with approximate ones estimated from objective functions. In case of stress and deflection functions, it is investigated that Chebyshev orthogonal polynomial is more useful than multiple regression polynomial. Although the coefficients of higher order terms are not known or there is much more difference between terms, Chebyshev orthogonal polynomial can normalize the base value and build regression model efficiently.

Table 4 Variance analysis of Chebyshev orthogonal polynomial for deflection

Design variable		Sum of squares	DOF	Variance	F-ratio	P-value
x_7	Linear	0.00139	1	0.00139 ○	4.3	0.174
x_1	Linear	0.10811	1	0.10811	335.34	0.003**
	Quadratic	0.00146	1	0.00146	4.52	0.167
x_2	Linear	0.00203	1	0.00203 ○	6.29	0.129
	Quadratic	0.00139	1	0.00139	4.32	0.173
x_3	Linear	1.17313	1	1.17313	3638.9	0**
	Quadratic	0.00277	1	0.00277	8.60	0.099
x_4	Linear	0.23773	1	0.23773	737.39	0.001**
	Quadratic	0.00037	1	0.00037	1.14	0.398
x_5	Linear	0.00007	1	0.00007 ○	0.22	0.687
	Quadratic	0.00045	1	0.00045	1.39	0.360
x_6	Linear	0.1133	1	0.1133	351.43	0.003**
	Quadratic	0.00043	1	0.00043	1.32	0.369
e	Linear	0	1	0 ○	0.01	0.932
	Quadratic	0.00001	1	0.00001	0.02	0.896
Error		0.00064	2	0.00032		
Total		1.64327	17			

○ : Pooling, ** : 1% Level of significance

Table 5 MAPE of polynomial regression expression

Degree	Mass	Stress	Deflection
1st polynomial	0.044%	7.501%	6.603%
2nd polynomial	9.629%	6.536%	6.057%

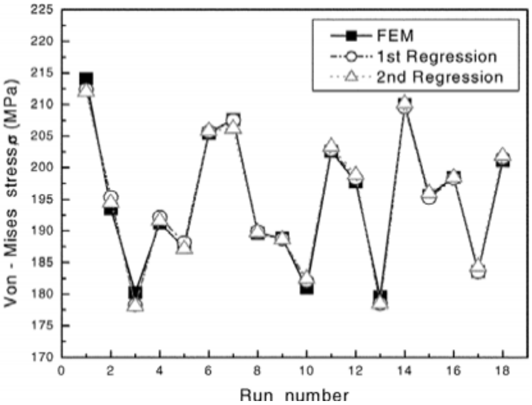

Fig. 5 Relation between Von-Mises stress and run number in DOE

These results are because it can maintain the independence between each term and have good regression capability only with partial lower order terms.

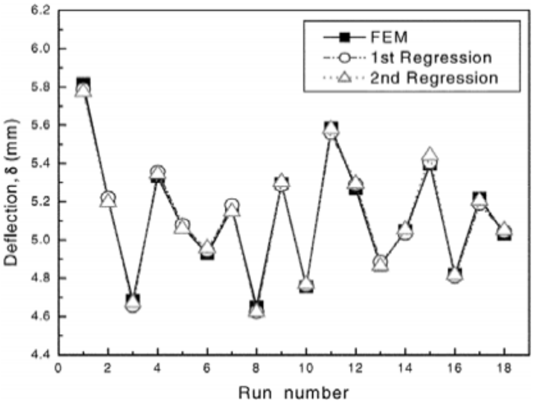

Fig. 6 Relation between deflection and run number in DOE

In case of mass functions, however, multiple regression polynomial which is affected by all the design variables has better approximation than Chebyshev orthogonal polynomial. In Figs. 5, 6 and 7 the comparison of FEA values and approximation values of objective functions is shown. The functions of response surface approximation are depicted in the following Eqs.

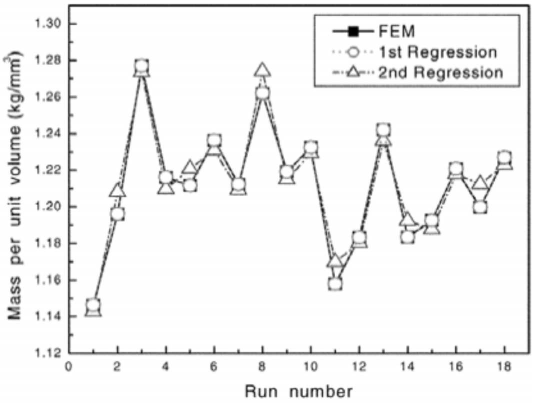

Fig. 7 Relation between mass per unit volume and run number in DOE

(8) ~ (10), respectively.

$$y_s = |\Delta l_{stress}| = 230.65 - 3.161x_4 + 0.5578x_4^2 - 11.6891x_5 - 0.09717x_7 \quad (7)$$

$$y_d = |\Delta l_{deflection}| = 6.498 - 0.0949x_2 - 0.418x_4 + 0.02633x_4^2 - 0.1406x_5^2 - 0.09717x_7 \quad (8)$$

$$y_m = |\Delta l_{totalmass}| = 1.080 - 0.015x_1 + 0.01229x_2 + 0.005458x_3 + 0.02338x_4 + 0.01824x_5 + 0.0002167x_6 + 0.01812x_7 \quad (9)$$

3.3 Problem formulation

In order to improve the safety and productivity of the mill, the objective functions of stress, deflection and mass are define and they are limited by predefined constraint conditions. The vertical roller mill is parameterized with 7 design variables. The multi-objective optimization of vertical roller mill under fatigue load in the standard mathematical format can be formulated as

(a) Objective function

Minimize $\{y_{stress}, y_{deflection}, y_{weight}\}$

$y_{stress} = 185.8$ MPa (Fatigue limit of the mill material), $y_{deflection} = 5.45$ mm, $y_{weight} = 1.2038$ kg/mm^3, where mass is defined as unit mass of axisymmetric model with thickness 1 mm for efficient optimization calculation.

(b) Constraint functions on the design variables

$1 \le x_1 \le 2$, $1 \le x_i \le 3$ $(i = 2, 3, \cdots, 7)$

3.4 Trade-off Analysis in multi-objective optimization

In multi-objective optimization problems, there exist trade-off functions between each objective functions. The minimization solution satisfying all the objective functions simultaneously, therefore, does not exits generally. Accordingly each objective function modifies the required objective and allowable limits which are changeable and then the optimization calculation is executed from the higher priority.

In case of performing optimization using SQP, there is always the priority between objective functions. It is supposed that the first, second and the third objective functions are related to the stress, the deflection and the mass respectively. Then SQP is used to find the optimum solution for the response surface model in an optimization process.

[1] First step

Minimize y_{stress} (10)

Subject to $y_{deflection} \le 5.45$

$y_{mass} \le 1.2038$

$1 \le x_1 \le 2$

$1 \le x_i \le 3$ $(i = 2, 3, \cdots, 7)$

[2] Second step

Minimize $y_{deflection}$ (11)

Subject to $y_{mass} \le 1.2038$

$y_{stress} \le 179.58 + 3.11$

$1 \le x_1 \le 2$

$1 \le x_i \le 3$ $(i = 2, 3, \cdots, 7)$

[3] Third step

Minimize y_{mass} (12)

Subject to $y_{stress} \le 179.58 + 3.11$

$y_{deflection} \le 5.2281 + 0.11095$

$1 \le x_1 \le 2$

$1 \le x_i \le 3$ $(i = 2, 3, \cdots, 7)$

In Table 6 the optimal solution at each optimization stage is shown. Among objective functions, only primary objective function is considered as one objective function and the others are expressed in the form of inequality constraint functions. First of all, the problem is solved as a single objective optimization problem and the

Table 6 Optimal solution

Step	Mass (kg/mm^3)	Stress (MPa)	Deflection (mm)
Initial	1.2038	227.343	6.254
First step	1.212	179.58	5.2299
Second step	1.2101	182.73	5.2281
Third step	1.2012	184.73	5.2904

optimum value Y^* of the required objective in the highest priority is obtained. The objective function obtained from the first optimization stage should be added to $\Delta(\Delta=|Y^*-Y^\varepsilon|/2)$ and reset for a constraint function at second optimization stage. This process is repeated until it converges within the given tolerance.

In Table 7 the values of design variables determined after 3 iterations of the optimizer are shown and in Fig. 8 the normalized objective functions determined during 3 optimization cycles are shown. Normalized objective function f is defined as follows.

$$f=\left\{\frac{|Y^\varepsilon-Y^*|}{Y^\varepsilon}\right\}+1 \quad (13)$$

Table 7 Optimal design variables

Result	x_7	x_1	x_2	x_3	x_4	x_5	x_6
Optimum level	1	1.0038	1	1.3633	3	1	1.745
Optimum size	Initial	0.06	−10	−32.7	40	−39	14.9

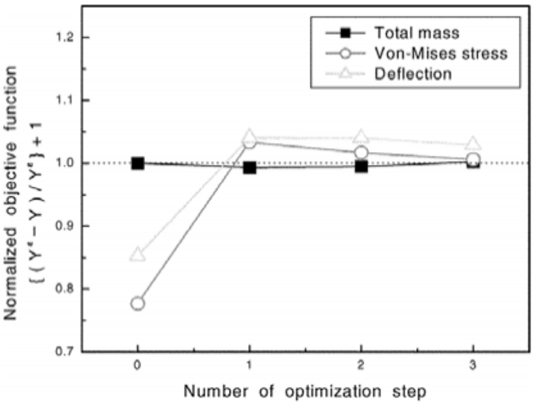

Fig. 8 Values of normalized objective function in the process of mill optimization with SQP

Table 8 Reanalysis solution

Result	Weight (kgf)	Stress (MPa)	Deflection (mm)
Initial	129,764	227.343	6.254
Optimum	128,486	184.731	5.291
Reanalysis	128,978	185.537	5.214

Here Y^* is the optimum value at each stage and Y^ε, is the required objective value. If normalized objective function exceeds 1, the objective function satisfies the constraints. At the initial design stage, stress and deflection do not exist in a feasible design space except for mass. The reason is that the relative importance between objective functions is not estimated quantitatively and there is much reliance on the intuition of the designer. Final optimization results shows that all the objective functions achieve the target values of the constraints. In Table 2 and Fig. 8, the best optimal design solution by SQP is much more superior to the design solution only by DOE. The results verified by FEA are compared with those predicted by fitted model in Table 8.

The predicted optimal solutions are also verified by FEA solutions within tolerance error. The difference between the verified and the predicted objective functions is due to the error from structural analysis and approximation. Obviously, the vertical roller mill optimized using the proposed DOE, RSM and SQP approach has better performance than the primitive design model.

This approach enables designer to have the immediate feedback suggestions for design improvement. Therefore it is considered that with the design model presented in this study, it is possible that a high performance product design can be achieved during the early design stage.

4. Conclusions

The RSM coupled with DOE and structural analysis provides an efficient design space for vertical roller mill. The statistical significance of design variables is estimated by F-test from ANOVA results. The multiple regression polynomial of degree 1 is suitable for weight affected by all the design variables, however, Chebyshev orthogonal polynomial of degree 2 is suitable for stress and deflection affected by only particular design variables. The vertical roller mill optimized by DOE, RSM and SQP approach has better performance than the primitive design model.

Therefore it is considered that SQP could be effectively used to guide the optimization of vertical roller mill and achieve a robust and reliable design in a most efficient way.

References

Bramanti, A., Barba, P. D., Farina, M. and Savini, A., 2001, "Combining Response Surfaces and Evolutionary Strategies for Multi-objective Pareto-Optimization in Electromagnetics," *International Journal of Applied Electromagnetics and Mechanics*, Vol. 15, pp. 231～236.

Baek, S. H., Hong, S. H., Cho, S. S. and Joo, W. S., 2004, "Multi-objective Optimization in Discrete Design Space using RSM-Based Approximation," *The Third China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems (CJK-OSM3)*, Japan, pp. 125～130.

Chen, S., Yang, J., Mou, J. and Lu, Y., 2002, "Quality Based Design Approach for a Single Crystal Silicon Microactuator using DOE Technique and Response Surface Model," *Microsystem Technologies*, Vol. 8, pp. 182～187.

Elsayed, E. A. and Chen, A., 1993, "Optimal Levels of Process Parameters for Products with Multiple Characteristics," *Int. J. Prod. Res.*, Vol. 31, No. 5, pp. 1117～1132.

Gautschi, W., 1996, "Orthogonal Polynomials : Application and Computations, Acta Numerica," *Cambridge University Press*, pp. 45～119.

Gunawan, S. and Azarm, S., 2005, "Multi-objective Robust Optimization using a Sensitivity Region Concept," *Structural and Multidisciplinary Optimization*, Vol. 29, No. 1, pp. 50～60.

Kurtaran, H., Eskandarian, A., Marzougui, D. and Bedewi, N. E., 2002, "Crashworthiness Design Optimization using Successive Response Surface Approximations," *Computational Mechanics*, Vol. 29, pp. 409～421.

Pignatiello, J. J., 1993, "Strategies for Robust Multi Response Quality Engineering," *IIE Trans.*, Vol. 25, No. 3, pp. 5～14.

Park, S. H., 1996, "Robust Design and Analysis for Quality Engineering," Chapman & Hall.

Roux, W., Stander, N. and Haftka, R., 1998, "Response Surface Approximations for Structural Optimization," *Int. J. Numer. Methods Eng.*, Vol. 42, pp. 517～534.

Redhe, M., Giger, M. and Nilsson, L., 2004, "An Investigation of Structural Optimization in Crashworthiness Design using a Stochastic Approach-A Comparison of Stochastic Optimization and the Response Surface Methodology," *Structural and Multidisciplinary Optimization*, Vol. 27, No. 6, pp. 446～459.

Tsui, K. L., 1999, "Robust Design Optimization for Multiple Characteristic Problems," Int. J. Prod. Res., Vol. 37, No. 2, pp. 433～445.

Timothy, W. S., Timothy, M. M., John, J. K. and Farrokh, M., 2001, "Kriging Models for Global Approximation in Simulation-Based Multidisciplinary Design Optimization," *AIAA Journal*, Vol. 39, No. 12, pp. 2233～2241.

Tappeta, R. V. and Renaud, J. E., 2001, "Interactive Multiobjective Optimization Design Strategy for Decision Based Design," *ASME J. Mech. Des.*, Vol. 123, pp. 205～215.

Wilson, B., Capperelleri, D., Simpson, T. W. and Frecker, M., 2001, "Efficient Pareto Frontier Exploration using Surrogate Approximations," *Optimization and Engineering*, Vol. 2, pp. 31～50.

Youn, B. D. and Choi, K. K., 2004, "A New Response Surface Methodology for Reliability-Based Design Optimization," *Computers and Structures*, Vol. 82, Iss. 2-3, pp. 241～256.

Zhang, H., Gopal, M. M., Saxena, R. and Avula, X. J., 2003, "An Integrated Optimization System for Airbag Design and Modeling by Finite Element Analysis," *SAE-paper 2003-01-0506, SAE World Congress*, Detroit, Michigan.